MODELLING OF AORTIC DISSECTION WITH BETA RANDOM FIELDS & UNCERTAINTY PROPAGATION WITH A BAYESIAN VARIATIONAL AUTO-ENCODER

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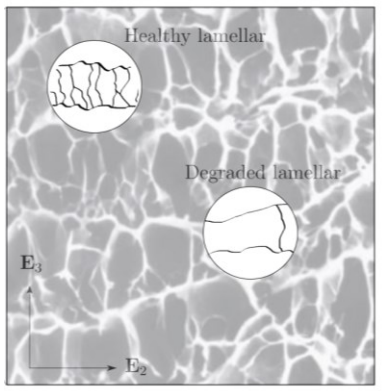
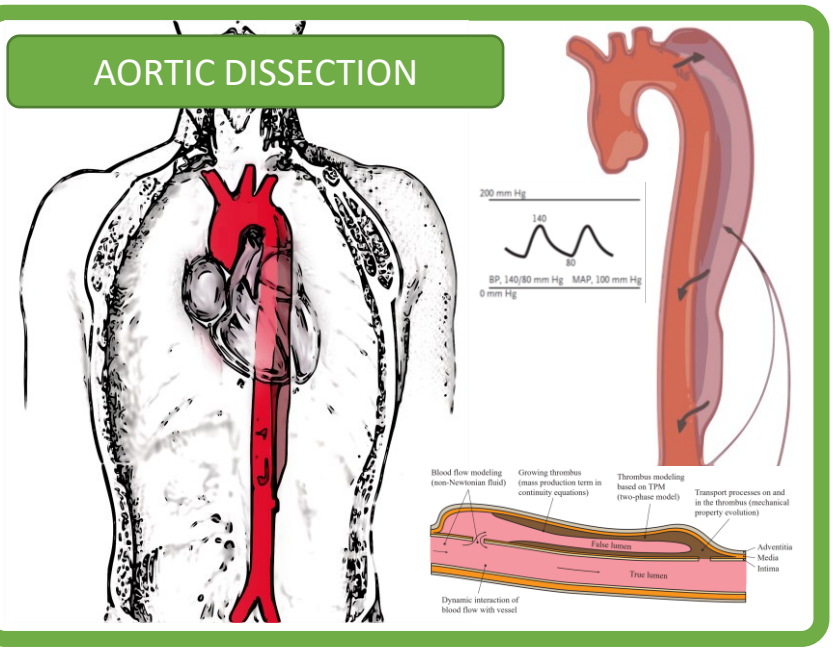
MICROSCOPE IMAGE



Material Parameter
$\in [0, 1]$

COMPUTER TOMOGRAPHY

AORTIC DISSECTION



Min. 4 Gaussian random field samples yield 1 Beta random field sample

BETA RANDOM FIELD

2D/3D-Gauss process:

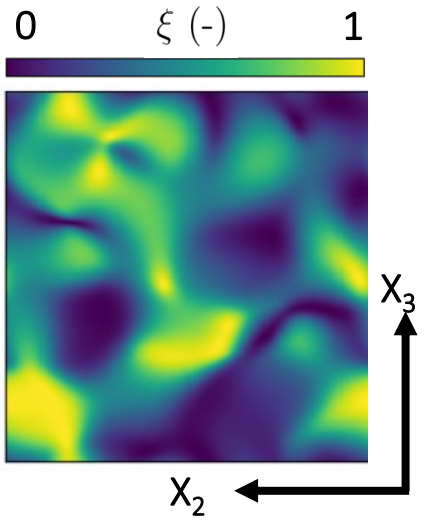
$$\mathcal{F} \sim \mathcal{GP}(\mu(\mathbf{X}), \mathrm{k}(\mathbf{X}, \mathbf{X}'))$$
$$\mathrm{k}(\mathbf{X}, \mathbf{X}') = \varsigma^2 \exp\left(-\frac{(\mathbf{X}-\mathbf{X}')^2}{2\ell^2}\right)$$

Gamma random field:

$$\mathbf{g}_s(\hat{\mathbf{X}}) = \frac{1}{2}\sum_{r=1}^{2s} \mathbf{f}_r^2(\hat{\mathbf{X}})$$

Beta random field:

$$\beta_{s,s'}(\hat{\mathbf{X}}) = \frac{\mathbf{g}_s(\hat{\mathbf{X}})}{\mathbf{g}_s(\hat{\mathbf{X}}) + \mathbf{g}_{s'}(\hat{\mathbf{X}})}$$



SIMULATION OF MECHANICAL FORCES

Compute the 33-component of Cauchy stress tensor BUT each random field sample takes several minutes of Finite Element simulation. Hence, a surrogate is needed!

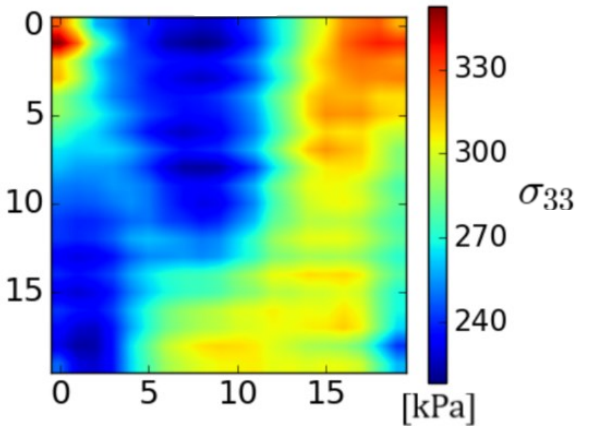


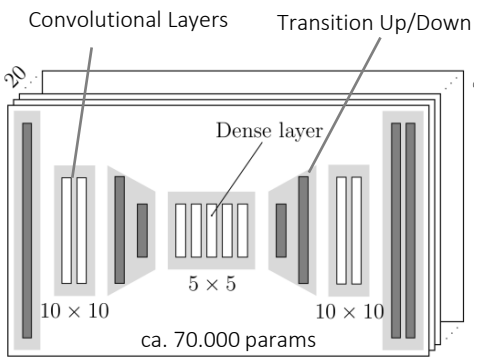
IMAGE-TO-IMAGE REGRESSION PROBLEM

BAYESIAN AUTO-ENCODER

Prior:
Student-T on network weights and biases

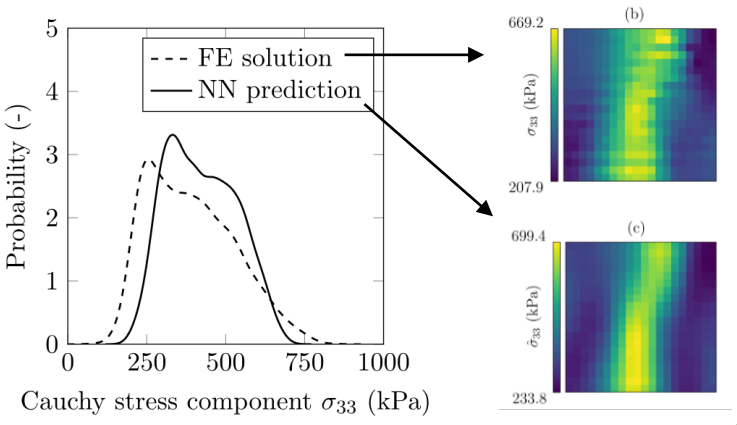
Likelihood:
Student-T with independent „pixels“

Training:
Stein Variational Gradient Desecent: Joint training of network ensemble allows approx. marginalization of network weights with non-parametric variational inference



RESULTS

Samples:



CONCLUSION

Bayesian Variational Auto-Encoders are also suitable for uncertainty propagation of finite element simulations of stochastics heterogeneous materials. Small scale fluctuations are troublsome to learn, but likelihood evaluation is 10^6 faster

References:
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[3] CT-scan: https://basicmedicalkey.com/aorta-ct
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[6] Rolf-Pissarczyk et al. 2020, doi: 10.1016/j.cma.2020.113511